Convolutional models

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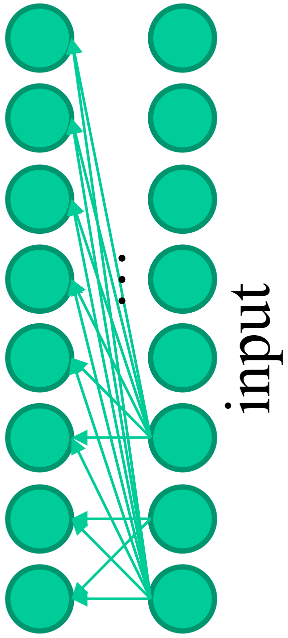
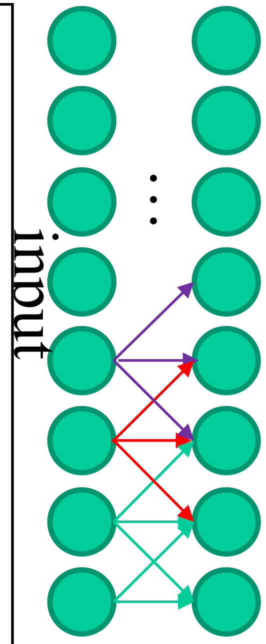
Local connectivity

- Sometimes, it might make sense to have local connectivity in a network





Why local connectivity?

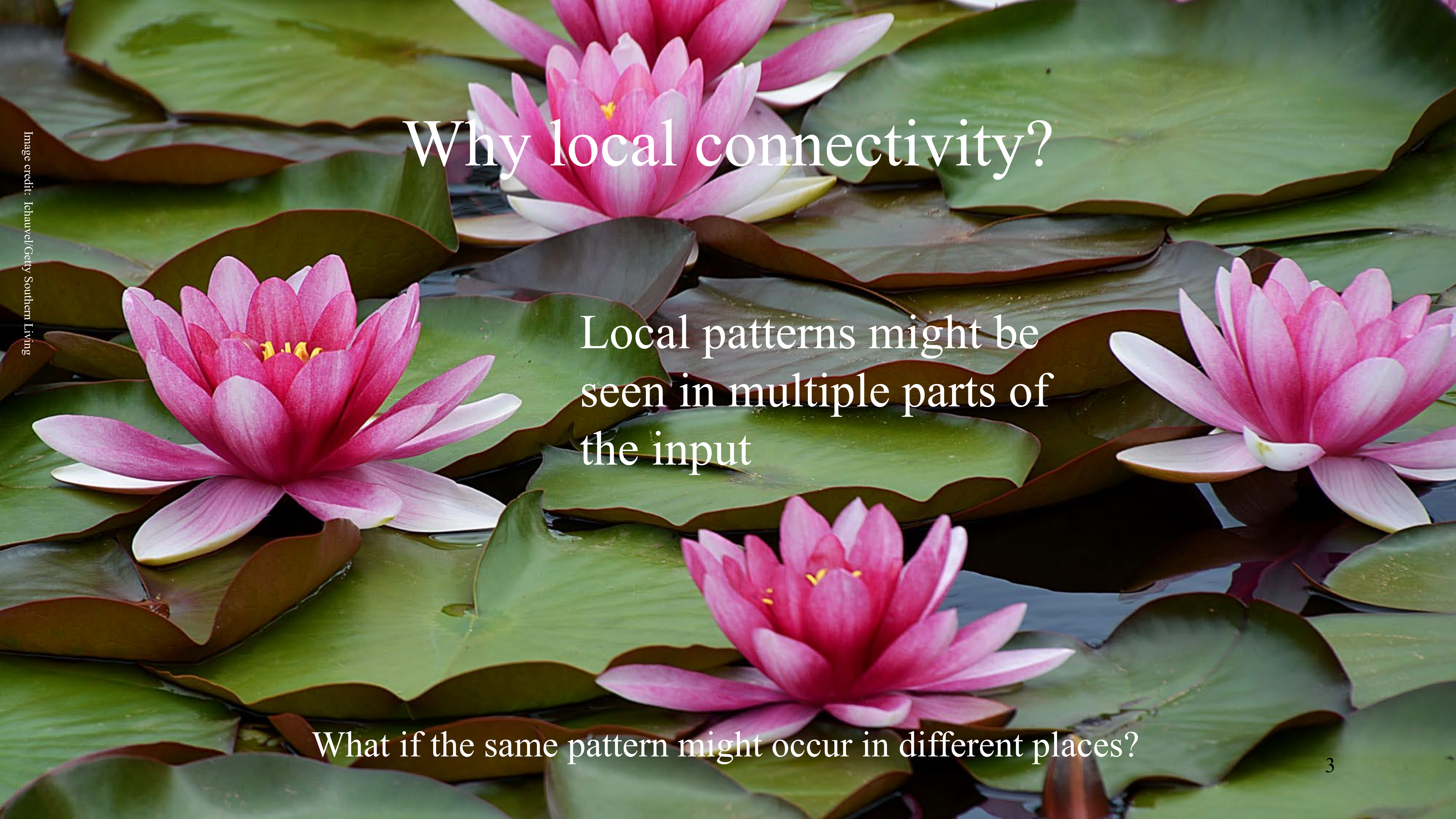


Image credit: lchauvel/Getty Southern Living

Local patterns might be seen in multiple parts of the input

What if the same pattern might occur in different places?

Local patterns

- How can we learn to recognize a local pattern anywhere in the input?
- If we can learn a set of weights that are applied to a local area, this can serve as a feature extractor.
- We need tools to apply these weights across the input.

Convolution

- Operation that combines two signals (1D definitions):
  – Continuous

$$x(t) * w(t) = \int_{-\infty}^{\infty} x(a)w(t-a)da$$

  – Discrete

$$x[t] * w[t] = \sum_{-\infty}^{\infty} x[a]w[t-a]$$

Notes on notation:
Engineers denote continuous signals with ( ), discrete with [ ]
Goodfellow et al. use (x*w)(t) rather than x(t)*w(t)

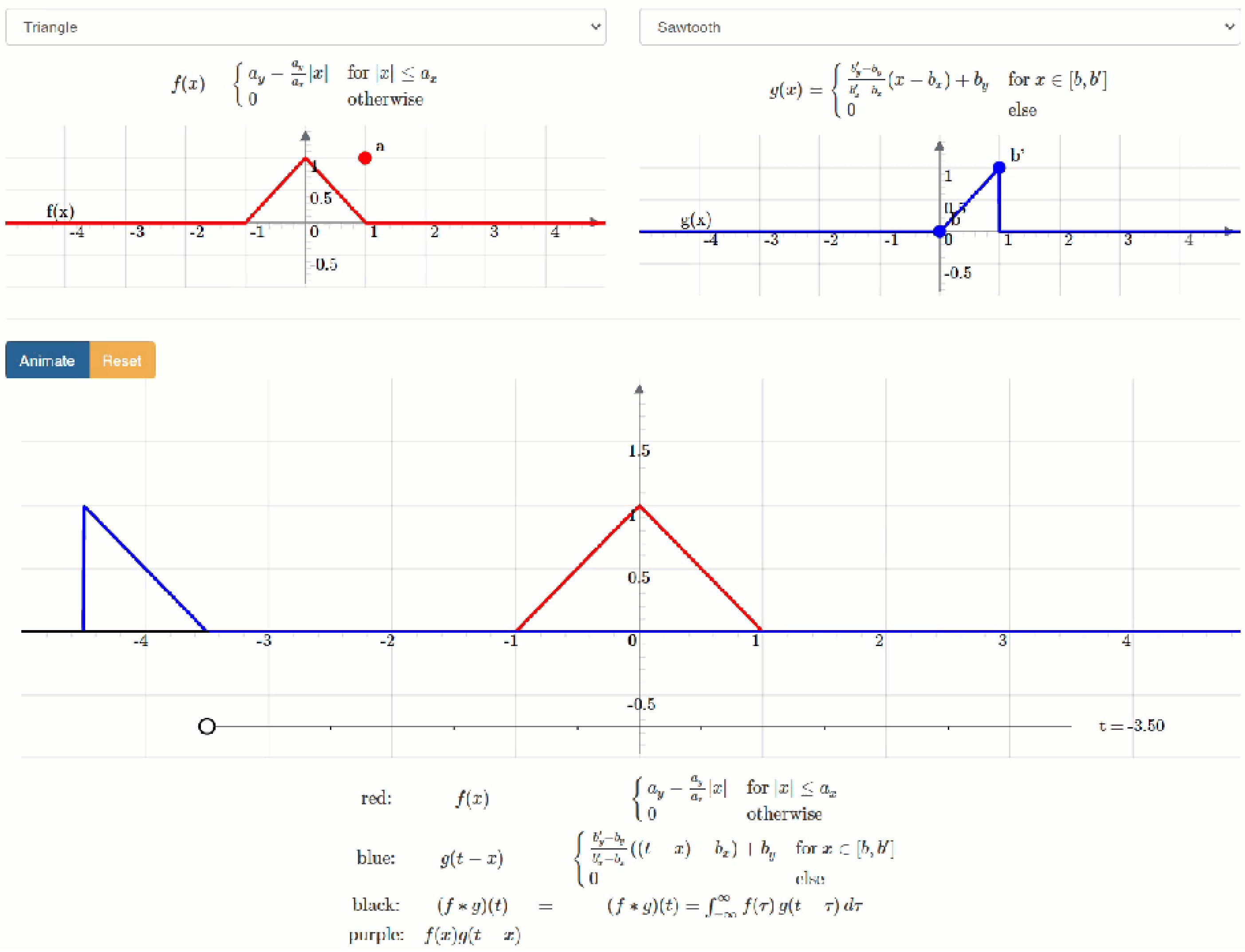
Convolution

- Formula outputs sample at time t.
- By convention:

$$\underbrace{x[t]}_{input} * \underbrace{w[t]}_{\substack{kernel \\ (filter)}} = \sum_{a=-\infty}^{\infty} x[a]w[t-a]$$

- The kernel shapes the response of the convolution and is time reversed.
- Kernel usually has finite support

Convolution in 1D



Demonstration courtesy of Karlsruhe Institute of Technology
https://phiresky.github.io/convolution-demo/

Convolution in 2D

- For audio, typically used on spectrograms or other time/frequency representations:

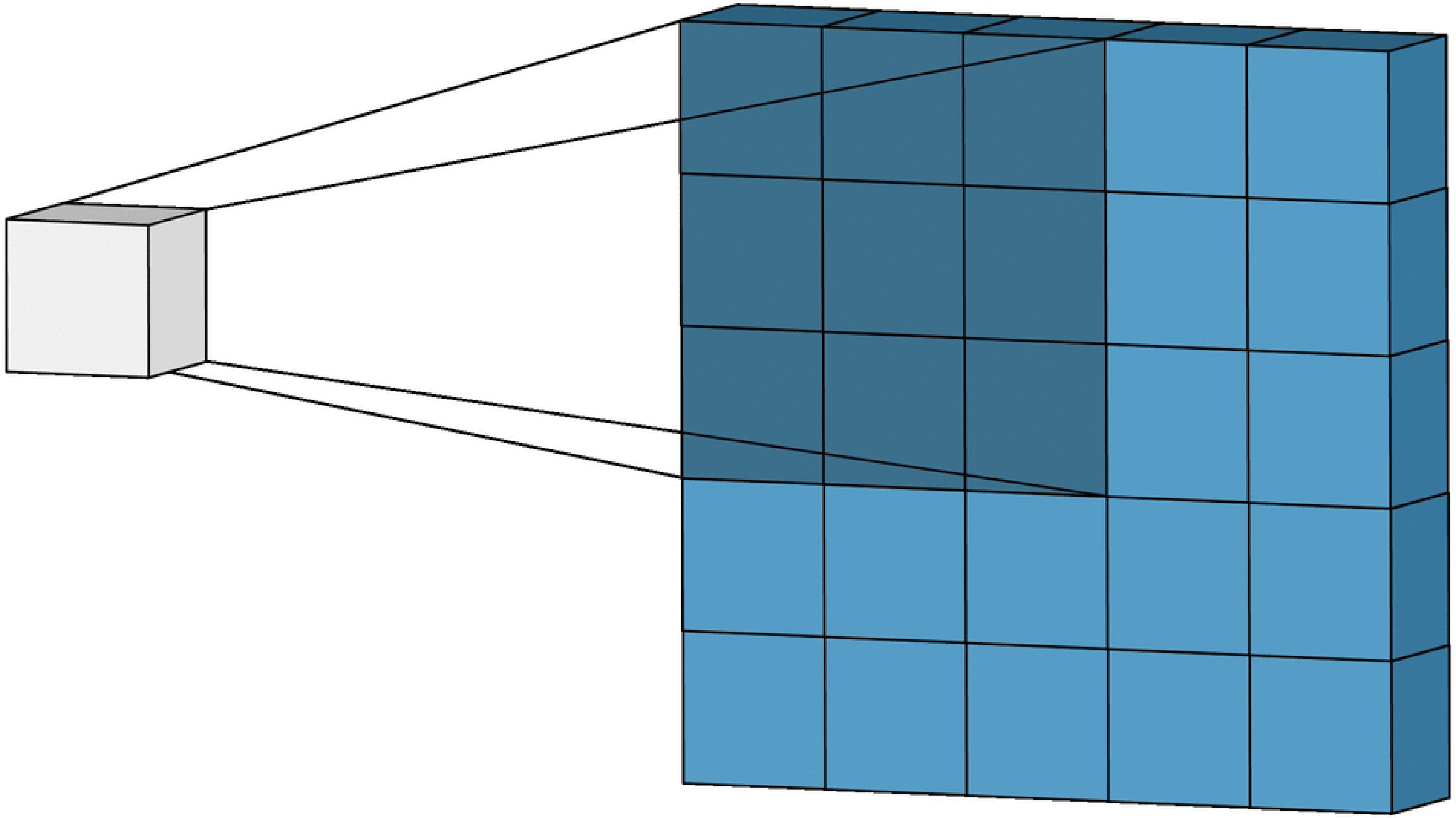
$$x[t,\omega] * w[t,\omega] = \sum_m \sum_n x[m,n]w[t-m,n-\omega]$$

- Many libraries implement cross-correlation instead of convolution (kernel not reversed, not important in practice):

$$\sum_m \sum_n x[t+m,\omega+n]w[m,n]$$

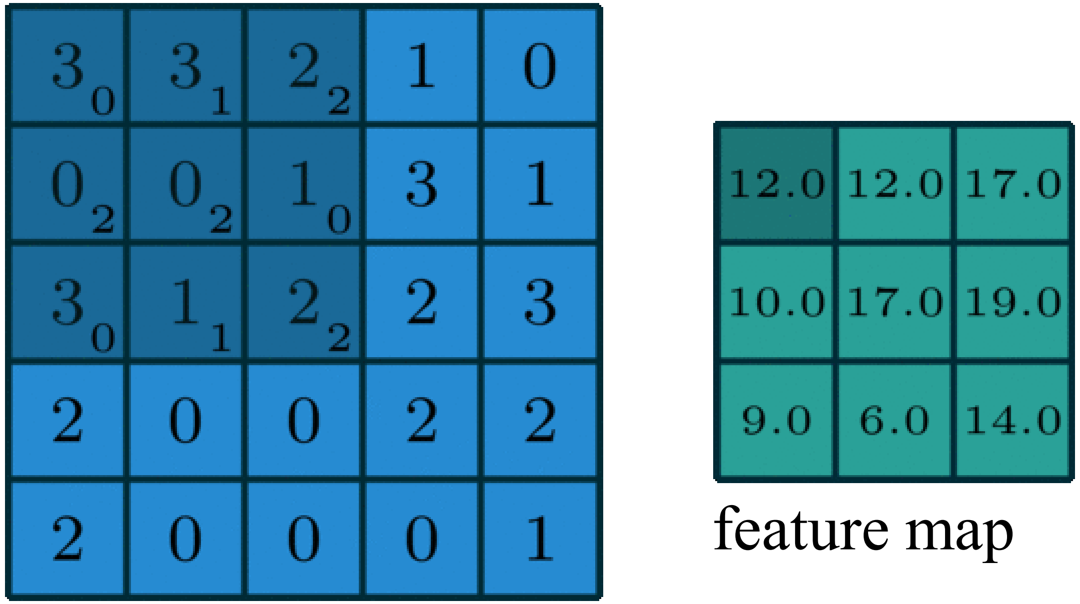
- Convolution is applied for each time-frequency cell $[t,\omega]$

Convolution in 2D



Animation credit: Irhum Shafkat

Convolution in 2D



feature map

$$w = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

Multiple kernels

- Suppose > 1 kernel, e.g., 10
- In the previous example, we would have 10 3x3 feature maps
- If the next layer convolved another 10 kernels across each of the 10 feature maps, we would have 100 feature maps.
- To prevent combinatorial explosion, we fuse the feature maps between layers, usually by addition, possibly after using an activation function

Key ideas

- Kernel is a learned feature extractor

- Desirable properties
  - Sparse interactions
  - Parameter sharing
  - Equivalence under translation

Sparse interaction

- Output points only depend on a local neighborhood.
- Similar to cutting connections (weights to zero) in a dense layer

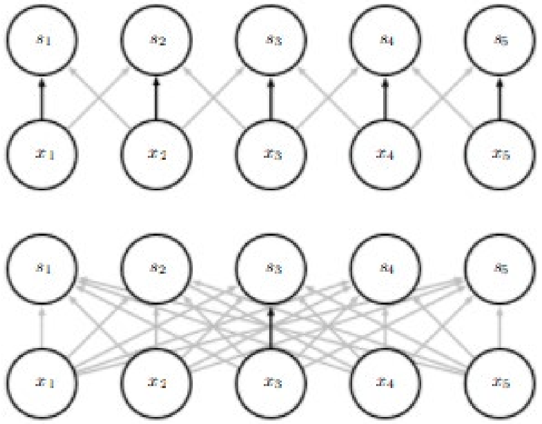


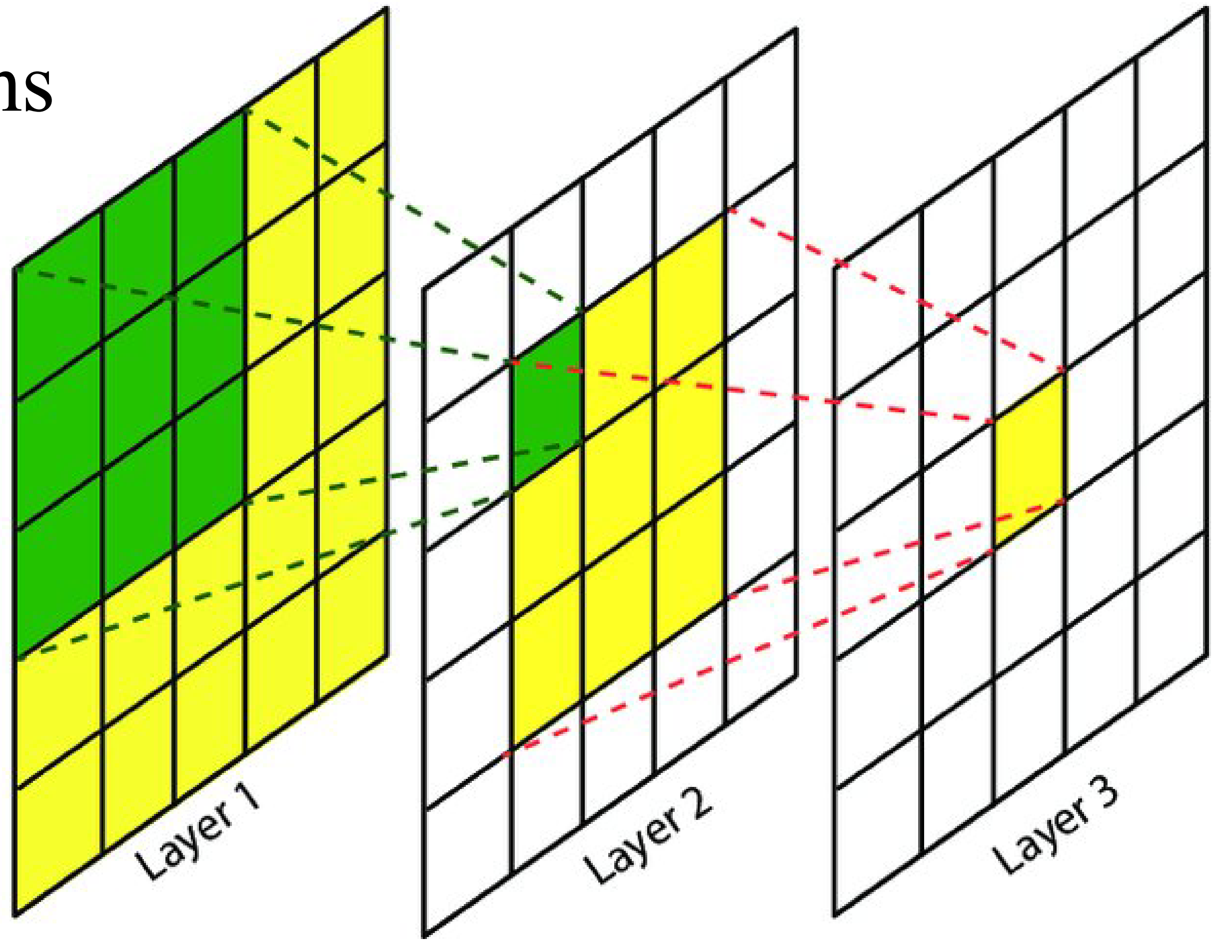
Fig. 9.2 Goodfellow et al.

Parameter sharing

- We slide the convolutional layer across the tensor.
- Each position produces a new output.
- We use the same kernel, thus the outputs from a single kernel have tied (shared) weights.

Receptive field

Subsequent convolutions increase the area that affects a layer output



Lin et al 2017 doi:10.3390/rs9050480

Equivariance under translation

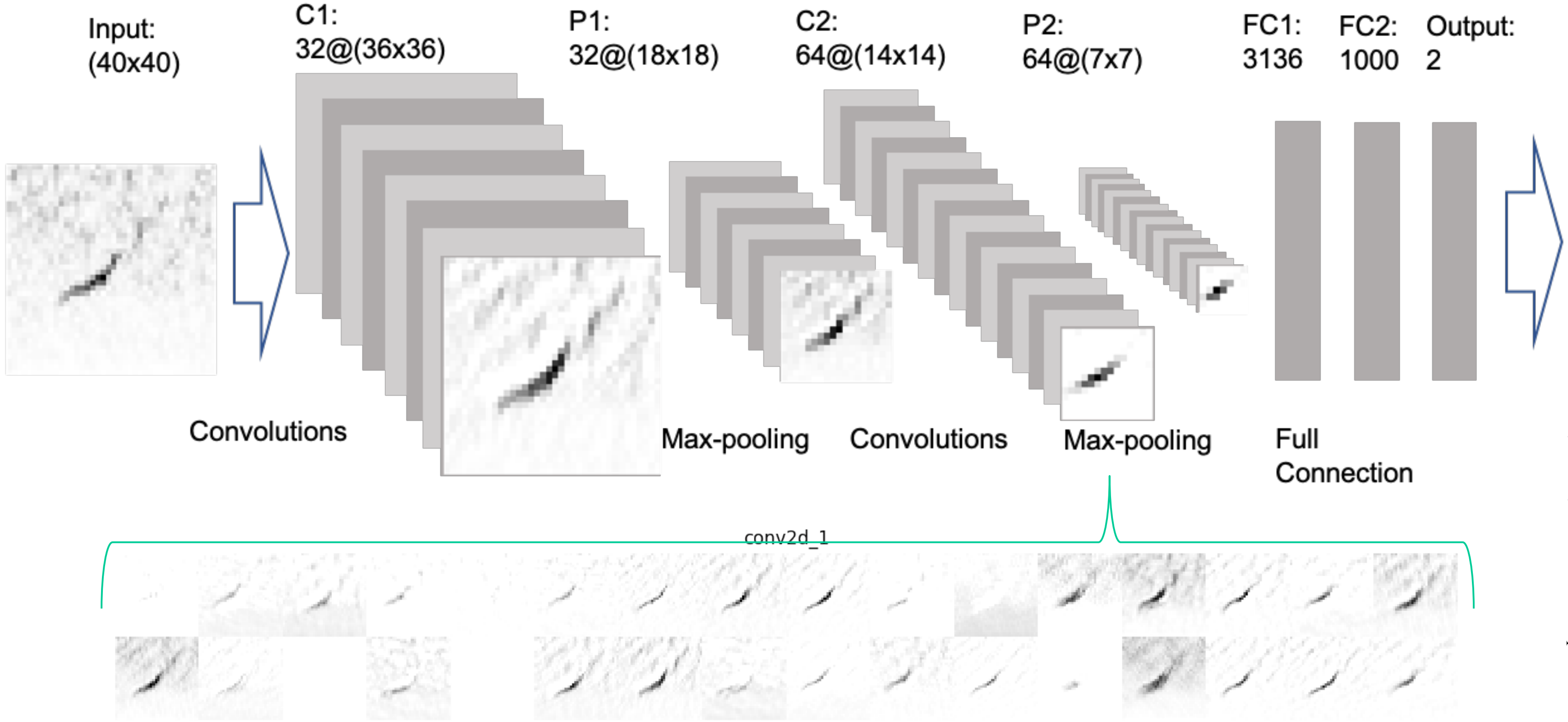
- When something in the input shifts position, its representation (convolution with the kernel) will also shift position.

Image: John Tlumacki, Boston Globe

image: NOAA

Key ideas

Convolutional layers learn a representation



Shiu, Palmer, et al. 2019 *Sci Rep*

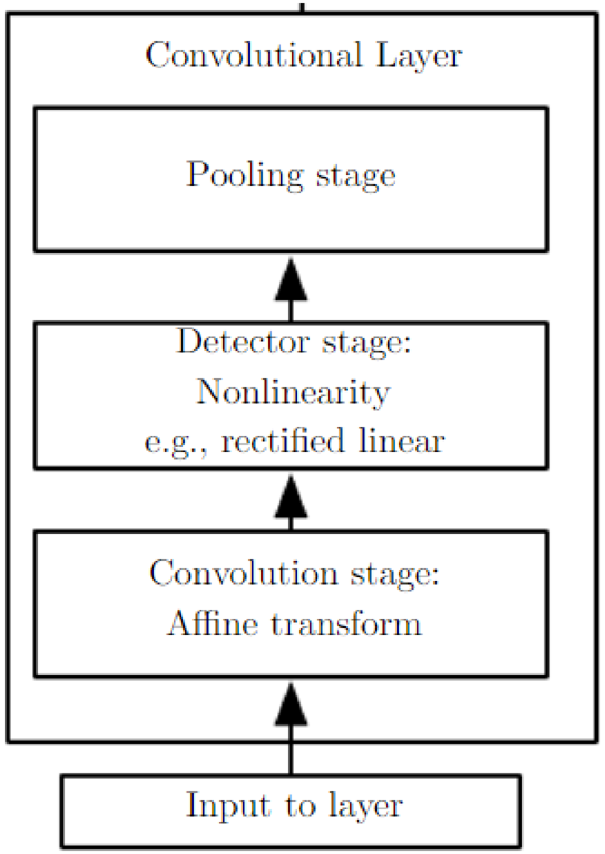
North Atlantic right whale upcall

Key ideas

- For audio, convolutional layers frequently followed by
  - an optional RNN
  - flattening and a feed-forward network to perform the classification.

Common architecture

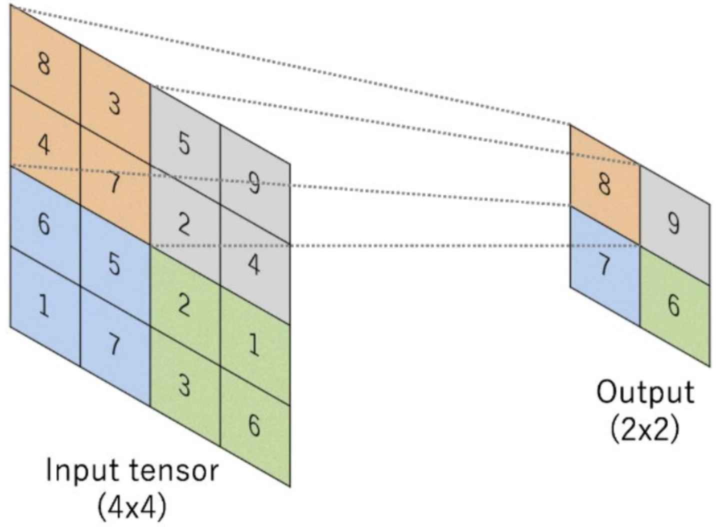
- Linear activation in first stages (not all networks use this)
- Nonlinear activations
- Pooling



Goodfellow et al 9.7

Pooling layers

- Reduce dimensions
- Helps with translation invariance
- How it works:
  - specify mask size (2x2)
  - specify stride (2)
  - apply statistic to covered cells
  - Example is max-pooling



Yamashita et al. 2018
https://doi.org/10.1007/s13244-018-0639-9

Pooling

- Can be seen as an alternative to resizing an input to a fixed size
- Example:
  - Learn a 9x9 feature vector to send to classification layer
  - Can adjust the pooling step sizes so that end result is 9x9 for many input sizes.

Convolutional layer connections

- Each convolutional layer l has l_f filters $f_l(i), 1 \le i \le l_f$.
- In layer $l+1$, most deepnet libraries will combine the l_f filter outputs with each of $l+1$'s convolutional filters:

$$fout_{l+1}(i) = b_{l+1}(i) + \sum_{j=1}^{l_f} fout_l(j) * f_{(l+1)}(i)$$

How are CNNs used in audio

- Two main paradigms
  - Image processing paradigm
    - Spectrogram
    - Multiple convolutional and pooling layers
  - Framed speech
    - 1D convolution on time domain or spectrum
- Possible RNN network on extracted feature matrix
- Feed forward layer

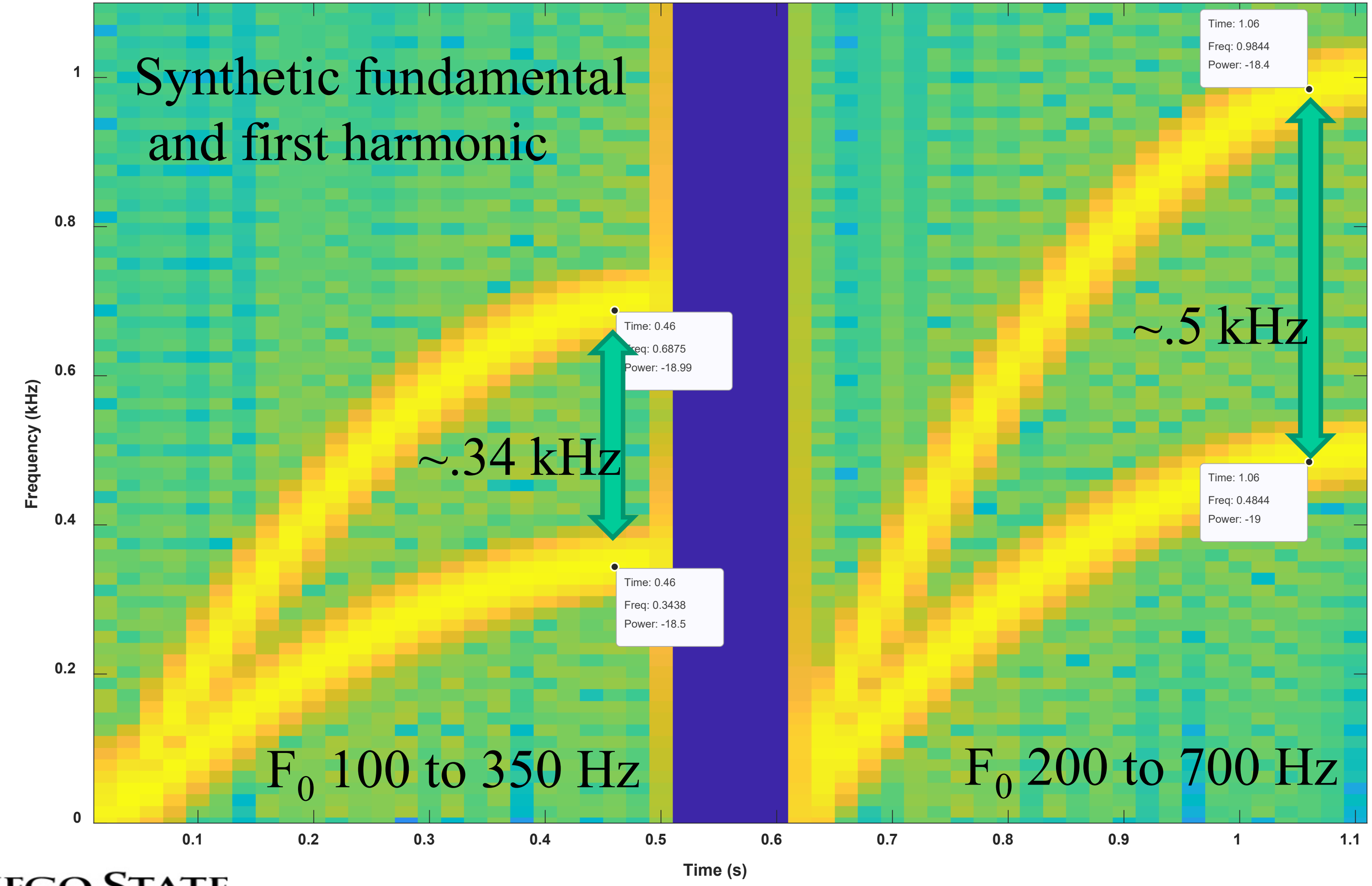
Are CNNs appropriate for audio?

Not always:

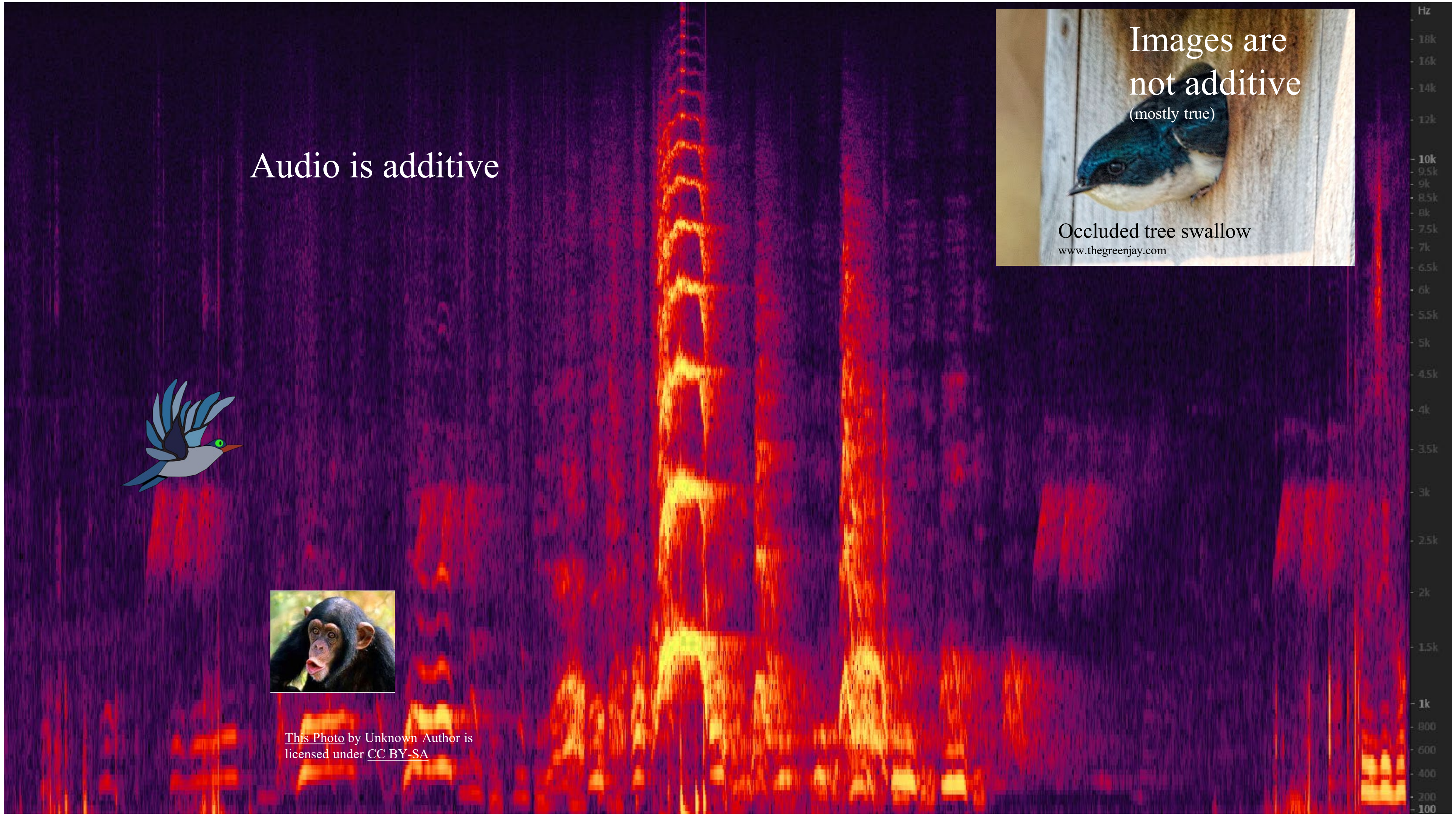
- Position has meaning

- Harmonic structure changes across the spectrum

Harmonic structure



San Diego State University



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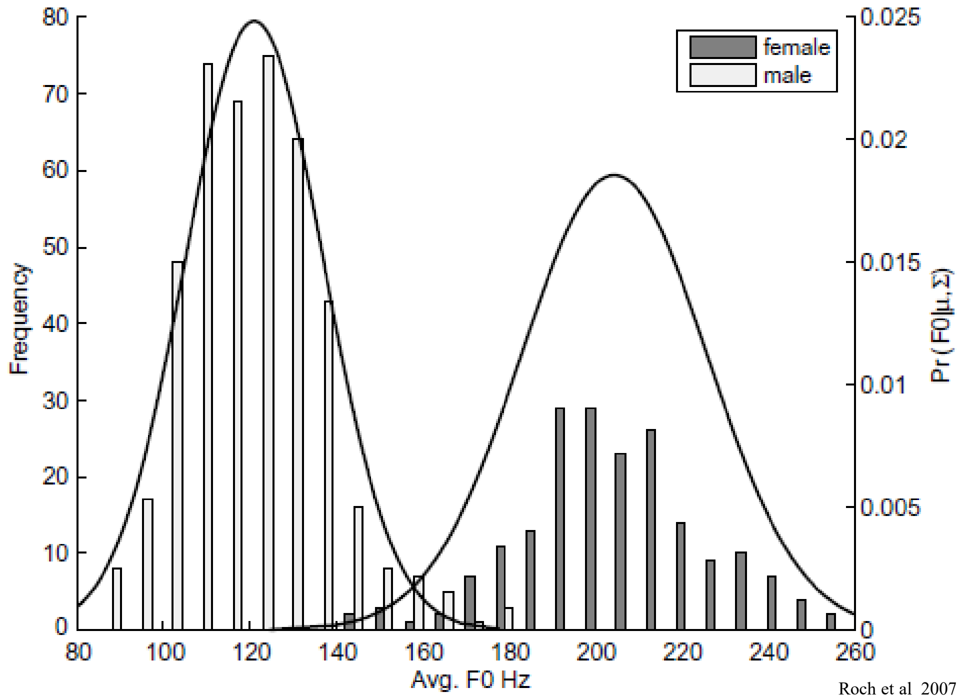
Positions within images are only
meaningful in relation to other things

Shifting frequency changes meaning…



Roch et al 2007

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Harmonic structure

- Unless using constant-quality representations harmonics change differently
- A CNN filter may learn harmonic structure that is appropriate for one frequency, but not for a frequency that a little different:
  - 100 200 300 400 Hz *vs*
  - 120 240 360 440 Hz

Keras convolutional layers

```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense, Input, Conv2D, MaxPooling2D
model = Sequential()
model.add(Input(shape=(1000,1000,1))  # 1000x1000 grayscale
model.add(Conv2D(32, kernel_size=(5, 5), activation='relu')))
model.add(MaxPooling2D(pool_size=(2, 2), strides=(2, 2)))
model.add(Conv2D(64, kernel_size=(5, 5), activation='relu'))
model.add(MaxPooling2D(pool_size=(2, 2)))
# Convert to vector, then feed forward network
model.add(Flatten())
model.add(Dense(1000, activation='relu'))
model.add(Dense(NumClasses, activation='softmax'))
```


Use Conv1D for 1D convolution