Assignment 5

Questions are worth 20 points each unless marked otherwise. Remember that quantitative questions must show work.

1. Consider the random variable *FavCandy* that represents the probability that a candy is someone's favorite candy. What is the conceptual difference between P(*FavCandy*=mounds) vs P(*FavCandy*=mounds | loves caramel)?

2. Use the product or chain rule to show how the joint probability of a series of stock prices on four consecutive days $P(S_4=35, S_3=33, S_2=31, S_1=34)$ can be represented as the product of 3 conditional probabilities and one prior probability.

Consider the following Bayes net model for surfing which is only loosely grounded in reality. In this boolean model, distant storms (S) and local wind (W) drive the presence of surf (U; surf's up!) which in turn affects the presence of clean surfable waves (C), groms (G; newbie surfers), and the potential to surf through the backdoor (B; type of surfing through the barrel of a wave)

P(S)	true	false
	.02	.98

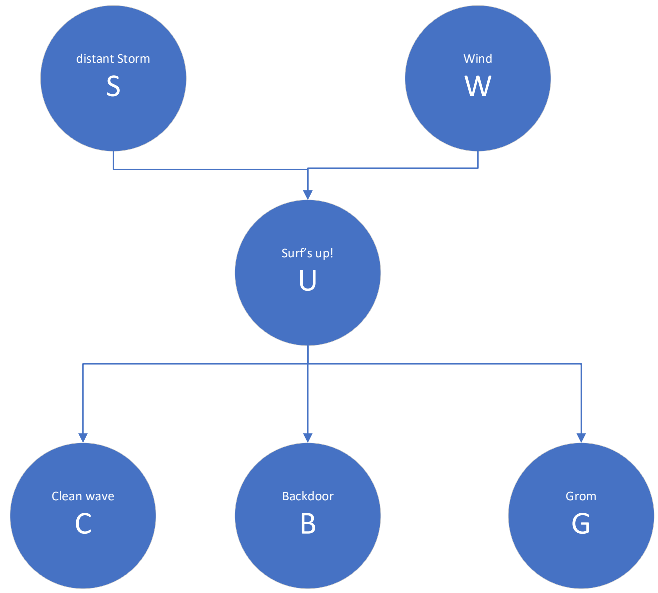
P(W)	true	false
	.55	.45

P(U|S,W)	true	false
false, false	.05	.95
false, true	.60	.40
true, false	.75	.25
true, true	.97	.03

P(C|U)	true	false
true	.5	.5
false	.2	.8

P(B|U)	true	false
true	.10	.90
false	.01	.99

P(G|U)	true	false
true	.15	.85
false	.80	.20



3. Compute the probability of a grom (newbie) surfer being present given that a distant storm is present and there is no local wind generating surf and surf's up being false.

4. Suppose that we know that a distant storm is present, but we do not know whether or not there is a local wind. Calculate the probability of a clean wave (good for surfing) occurring.

5. (40 points) Suppose that we have a set of three urns which contain colored balls. In a process unseen by an observer, an urn is selected, and then a colored ball is drawn from that urn. The observer only sees a sequence of balls. This can be modeled by a discrete HMM as follows:

   - $\pi = [.3 \quad .5 \quad .2]'$
   - $A = \begin{bmatrix} .5 & .10 & .40 \\ .25 & .5 & .25 \\ .30 & .20 & .5 \end{bmatrix}$
   - $b_1(x) = \begin{cases} .2 & x = red \\ .2 & x = yellow \\ .3 & x = blue \\ .3 & x = purple \end{cases}$, $b_2(x) = \begin{cases} .3 & x = red \\ .3 & x = yellow \\ .2 & x = blue \\ .2 & x = purple \end{cases}$,

     $b_3(x) = \begin{cases} .3 & x = red \\ .1 & x = yellow \\ .2 & x = blue \\ .4 & x = purple \end{cases}$

   Showing your work, compute the all path probability of the observation sequence: blue, yellow red.

6. (20 points) Write $P(O, q_t = s_i, q_{t+1} = s_j, q_{t+2} = s_k \mid \Phi)$ in terms of the forward and backward probabilities as well as probabilities that can be directly obtained from model Φ (e.g. $a_{i,j}$).