Mick Jagger, wordpress.com

I can't get no… Constraint Satisfaction

Professor Marie Roch

Chapter 6, Russell & Norvig

(skip 6.3.4, 6.5)

Constraint satisfaction problems (CSP)

Solutions with caveats

Example: Find a way to take classes such that I graduate in four years

- prerequisites
- course availability
- funding

Adobe Stock

Constraint satisfaction problems (CSP)

- To date, states were
  - atomic – didn't care about internal representation except with respect to analyzing for goal/heuristic
  - mutated by actions that produced a new atomic state

- Factored representations
  - states have internal structure
  - structure can be manipulated
  - constraints relate different parts of the structure to one another and provide legal/illegal configurations

CSP Definition

Problem = {X, D, C}

- X – Set of variables

$$X = \{X_1, X_2, \dots, X_n\}$$

- D – Set of domains
  such that

$$D = \{D_1, D_2, \dots, D_n\}$$

$$\mathrm{X}_i = x_i \text{ where } x_i \in D_i$$

- C – Set of constraints
  such that

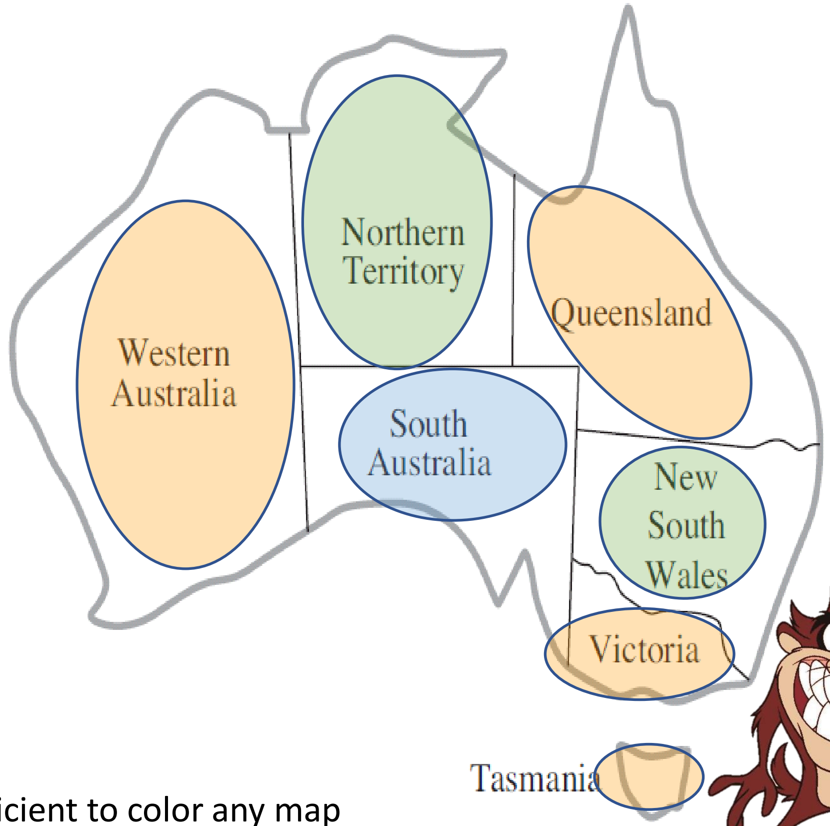
$$C = \{C_1, C_2, \dots, C_m\}$$

$$C_i = < (Variables), \text{relationship}(variables) >$$

Example: $< (X_2, X_5)\ ,\ X_2 \neq X_5 >$

CSP example: map coloring

- Color territories on a map using 3 colors such that no two colors are adjacent

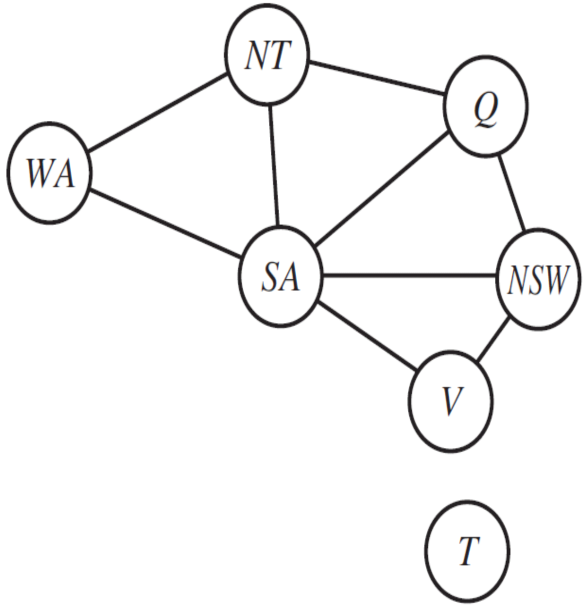


One possible solution for colors:
orange, blue, and green

Note: 4 colors are sufficient to color any map

Map coloring

- Graph representation
- Variables
  X={WA, NT, SA, Q, NSW, V, T}
- All variables have the same domain
  D_i={red, green, blue}
- Constraint set
  C={SA $\neq$ WA, SA $\neq$ NT, SA $\neq$ Q, SA $\neq$ NSW, SA $\neq$ V,
  WA $\neq$ NT, NT $\neq$ Q, Q $\neq$ NSW, NSW $\neq$ V}
  or {adjacent(t_a,t_b)$\rightarrow t_a \neq t_b$}



Scheduling example

Partial auto assembly

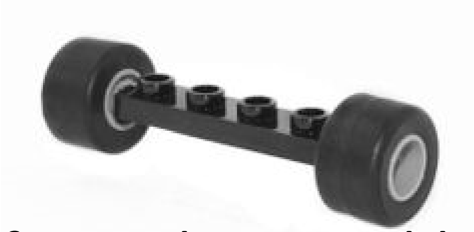
- Install front and rear axels (10 min each)
- Install four wheels (1 min each)
- Install nuts on wheels (2 min each wheel)
- Attach hubcap (1 min each)
- Inspect

Variable set X

$$X = \left\{ \begin{array}{cccc} Axle_F, & Axle_B, & Wheel_{RF}, & Wheel_{LF}, \\ Wheel_{RB}, & Wheel_{LB}, & Nuts_{RF}, & Nuts_{LF}, \\ Nuts_{RB}, & Nuts_{LB}, & Cap_{RF}, & Cap_{LF}, \\ Cap_{RB}, & Cap_{LB}, & Inspect & \end{array} \right\}$$

Constraint types

- Domain values
  - Time at which task begins {0, 1, 2, …}
- Precedence constraints
  - Suppose it takes 10 minutes to install axles.
  - We can ensure that front wheels are not started before axle assembly is completed:



$$Axle_F + 10 \leq Wheel_{RF}$$
$$Axle_F + 10 \leq Wheel_{LF}$$

- Disjunctive constraints – e.g. doohickey needed to assemble axle, but only have one

$$Axle_F + 10 \leq Axle_B \text{ or } Axle_B + 10 \leq Axle_F$$

Constraint types

- Unary – single variable $Z \leq 10$

- Binary – between two variables $Z^2 > Y$

- Global – constraints with 3+ variables
  can be reduced to multiple binary/unary constraints

$$X \leq Y \leq Z \rightarrow X \leq Y \text{ and } Y \leq Z$$

$$alldiff(W, X, Y, Z) \rightarrow W \neq X, W \neq Y, W \neq Z, X \neq Y, \ldots$$

Note: Global constraints do not have to involve *all* variables

Constraint graphs

Cryptoarithmetic puzzle
Find a different digit for each letter such that substitution results in a valid equation.

$$
\begin{array}{r}
T\ W\ O \\
+\ T\ W\ O \\
\hline
F\ O\ U\ R
\end{array}
$$

CSP specification

- $X = \{F, T, U, W, R, O, C_1, C_2, C_3\}$
- $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- $C = \{$

  $O + O = R + 10\ C_1$
  $C_1 + W + W = U + 10\ C_2$
  $C_2 + T + T = O + 10C_3$
  $C_3 = F$
  Alldiff(F,T,U,W,R,O)
  $\}$

C_i's are auxiliary variables for carry digits

Alldiff(X_1, X_2,…, X_i) → $\forall j, k\colon j \neq k\ and\ 1 \leq j, k \leq i, x_j \neq x_k$

Constraint hypergraphs

$$\begin{array}{r} T\ W\ O \\ +\ T\ W\ O \\ \hline F\ O\ U\ R \end{array}$$

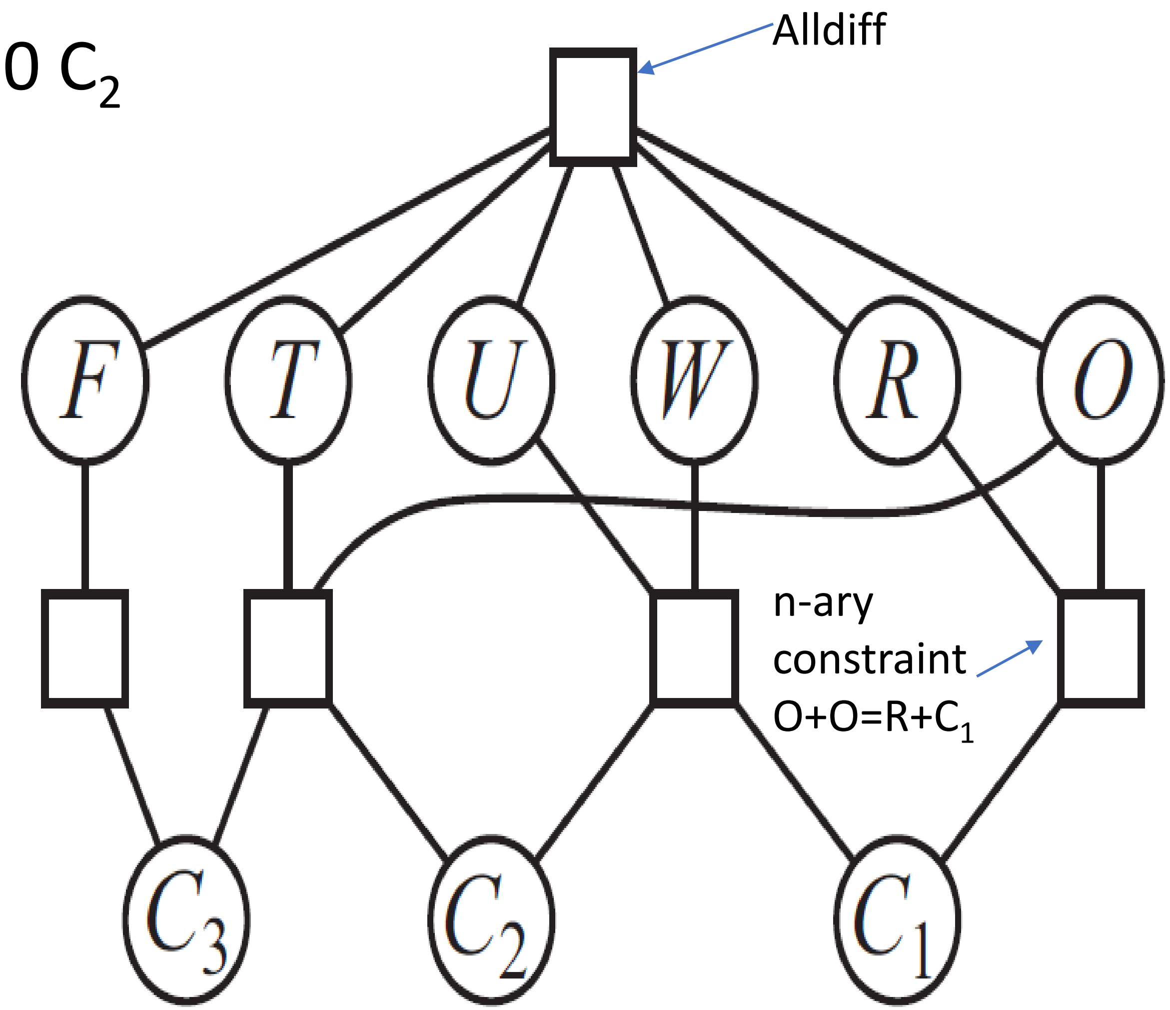
$O + O = R + 10\ C_1$

$C_1\ + W + W = U + 10\ C_2$

$C_2 + T + T = O + C_3$

$C_3 = F$

Alldiff(F,T,U,W,R,O)



Binarization of constraints

- Convert n-ary constraints into unary/binary ones.
- Example: constraint on X, Y, Z with domains:

$$X \in \{1,2\}\, Y \in \{3,4\}, Z \in \{5,6\}$$

- Create *encapsulated variable U*
  Cartesian product $U = X \times Y \times Z$

$$U \in \begin{Bmatrix} (1,3,5),(1,3,6),(1,4,5),(1,4,6), \\ (2,3,5),(2,3,6),(2,4,5),(2,4,6) \end{Bmatrix}$$

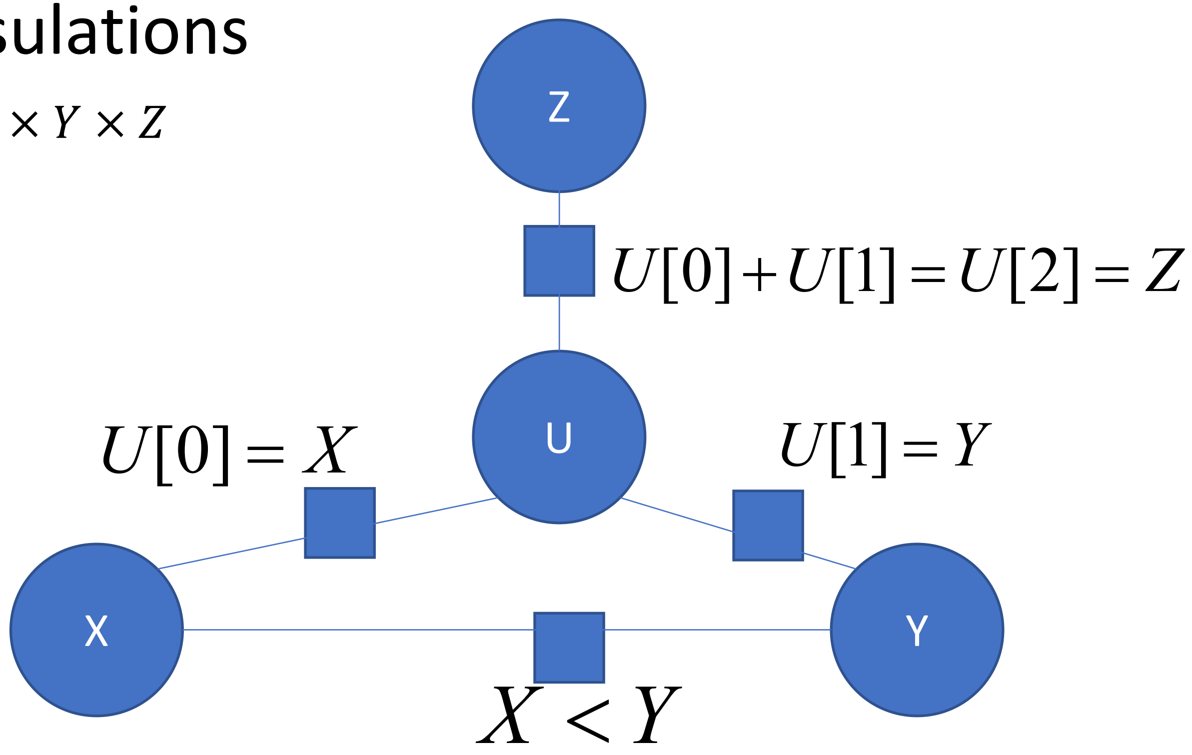
Equivalent binary CSP

- Constraints: $X + Y = Z$

  $X < Y$

- Encapsulations

  $U \triangleq X \times Y \times Z$



Another example: House puzzle

- A row of 5 houses, each one
  - has a color
  - contains a person with a nationality
  - has a household favorite candy
  - has a household favorite drink
  - contains a pet

  - all attributes are distinct

- How should we represent this?

House Puzzle Constraints

- The Englishman lives in the red house.
- The Spaniard owns the dog.
- The Norwegian lives in the first house on the left.
- The green house is immediately to the right of the ivory house.
- The man who eats Hershey bars lives in the house next to the man with the fox.
- Kit Kats are eaten in the yellow house.
- The Norwegian lives next to the blue house.
- The Smarties eater owns snails.

House Puzzle Constraints

- The Snickers eater drinks orange juice.
- The Ukranian drinks tea.
- The Japanese eats Milky Ways
- Kit Kats are eaten in a house next to where the horse is kept.
- Coffee is drunk in the green house.
- Milk is drunk in the middle house.

Answer the questions:

Where does the zebra live?

Which house drinks water?

House Puzzle Representation

- Variables – What’s common to each thing?

- Domains – What are the domains?

House Puzzle representation

- Constraints are location based, e.g. milk is drunk in the middle house.

- Could we associate variables with a location?

- If so, what are
  - our variables?
  - their domains?
  - and how do we write our constraints?

House puzzle representation

- Colors: red, green, ivory, yellow, & blue
- Nationalities: English, Spaniard, Norwegian, Ukranian, and Japanese
- Pets: dog, fox, snails, horse, and *zebra*
- Candies: Hershey bars, Kit Kats, Smarties, Snickers, and Milky Way
- Drinks: orange juice, tea, coffee, milk, and *water*

Note: water and zebra were inferred from the questions

House puzzle representation

Some examples

- Milk is drunk in the middle house.

  milk = 3
- Coffee is drunk in the green house

  coffee = green
- Kit Kats are eaten in a house next to where the horse is kept.

  abs(kit kats – horse) = 1
- The green house is immediately to the right of the ivory home.

  green = ivory + 1
- The Norwegian lives next to the blue house

  Norwegian = blue + 1 or Norwegian = blue – 1
- The Norwegian lives in the first house on the left

  Norwegian = 1

Implementing a CSP problem: Representation

- variables – simple list
- values – Mapping from variables to value lists
  e.g. Python dictionary
- neighbors – Mapping from variables to list of other variables that participate in constraints
- binary constraints
  - explicit value pairs
  - functions that return a boolean value

Representation of house problem

- variables:
  list of colors, nationalities, pets, candies, & drinks
  {red, green, ivory, yellow, blue, English, Spaniard, …}

- values: $X_i \in \{1,2,3,4,5\}$
  except milk = {3}, Norwegian = {1}

- neighbors:
  - all variable pairs from constraints, e.g. Englishman & red
  - alldiff(red, green, ivory, blue), alldiff(English, Spaniard, …), other category alldiffs

Representation of house problem

- constraints – Function f(A, a, B, b)

  where A and B are variables with values a and b respectively.

  Returns true if constraint is satisfied, otherwise false

  Example: f("Englishman", 4, "red", 5) returns false as the Englishman lives in the red house.

Let’s think about inference

- We know: Norwegian = {1}, milk = {3}
- Consider:
  Norwegian = blue + 1 or Norwegian = blue – 1

  with a sprinkle of algebra we have:
  blue = Norwegian – 1 or blue = Norwegian+1
  or as Norwegian can only be 1:
  blue = 1-1 or blue = 1+1

- We know blue ∈{1,2,3,4,5} , therefore blue=2
- Due to the alldiff constraint, we also know:

$$\forall color_{color \neq blue} color \in \{1,3,4,5\}$$

$$\forall natl_{natl \neq Norwegian} natl \in \{2,3,4,5\}$$

Another inference example

- We have the constraint: green = ivory + 1
  and we know that green & ivory in {1,3,4,5}

- Suppose green=3, can the constraint hold?
- What about green=1?
- We can deduce: $green \in \{4,5\}$
- What about ivory?

How do we formally tame this beastie?

Image: Rankin/Bass 1964

General strategies

- *Local consistency*: Reduce set of possible values through constraint enforcement and propagation
  - node consistency
  - arc consistency
  - path consistency
- Perform search on remaining possible states

Node consistency

- A variable is *node-consistent* if all values satisfy all unary constraints

$$fruits = \left\{ \begin{matrix} apples, oranges, strawberries, \\ peaches, pineapple, bananas \end{matrix} \right\}$$

Condition: $allergic(TreeBornFruit)$

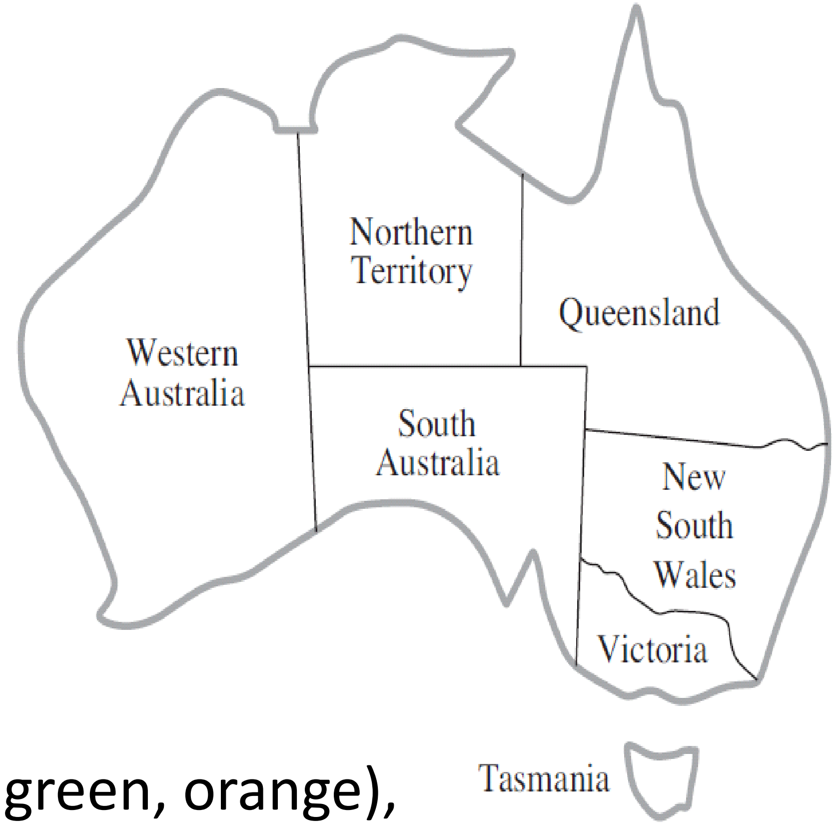
Reduced domain: $\{strawberries, pineapple\}$

- Other unary conditions could further restrict the domain

Arc consistency

- *arc-consistent*
  - variable - all binary constraints are satisfied for the variable
  - network – all variables in CSP are arc-consistent
- Arc consistency only helps when some combinations of values preclude others…

Arc Consistency



Each territory has domain
{orange, green, blue}

WA $\neq$ SA:
{(orange, green), (orange, blue), (green, orange),
(green, blue), (blue, orange), (blue, green)}

Does this reduce the domain of WA or SA?

Arc Consistency

- Constraints that eliminate part of the domain can improve arc consistency
- Variables that represent task starting times

T1 = {0, 1, 2, 3, 4, 5, 6, 7 , 8, 9}

T2 = {2, 3, 4, 5, 6, 7, 8, 9}

- Constraint: T1 + 5 < T2 yield consistent domains

T1 = {0, 1, 2, 3, 4}

T2 = {6, 7, 8, 9}

AC-3 arc consistency algorithm

```
AC3(CSP):
  "CSP(variables X, domains D, constraints C)"
  q = Queue(binary arcs in CSP)
  while not q.empty():
    (Xi, Xj) = q.dequeue()  # get binary constraint
    if revise(CSP, Xi, Xj):
       if domain(Xi) = ∅ return False
       else:
        for each Xk in {neighbors(Xi)- Xj}:
           q.enqueue(Xk, Xi)
  return True
```

$O(cd^3)$ worst case complexity (c # constraints, d max domain size)

AC-3 arc consistency

```
revise(CSP, Xi, Xj):
  “Restrict domain Xi such that it is consistent with Xj”
  revised = False
  for each x in domain(Xi):
    if not ∃y∈domain(Xj) such that
           constraint holds between x & y:
     delete x from domain(Xi)
     revised = True

  return revised
```

Note: In your text, D_i is used instead of domain(X_i). They are the same thing and both represent the domain values that have not yet been eliminated. This is the *current* domain of the variable, not the original one.

Path and k- consistency

- Higher levels of consistency, beyond our scope

- General ideas:
  - Path consistency
    See if a pair of variables $\{X_i, X_j\}$ consistent with a 3rd variable X_k. Solved similarly to arc consistency

  - K-consistency
    Given k-1 consistent variables, can we make a k^{th} variable consistent (generalization of consistency)

Global constraints

Consider the “all different” constraint.

- Each variable has to have a distinct value.
- Assume m variables, and n distinct values.
- What happens when $m > n$?

Global constraints

Extending this idea:

- Find variables constrained to a single value
- Remove these variables and their values from all variables.
- Repeat until no variable is constrained to a single value
- Constraints cannot be satisfied if
    1. A variable remains with an empty domain
    2. There are more variables than remaining values

Resource constraints ("atmost")

$$atmost(20, X, Y, Z) \rightarrow X + Y + Z \leq 20$$

$$atmost(10, P_1, P_2, P_3, P_4) \rightarrow \sum_{i=1}^{4} P_i \leq 10$$

- Consistency checks
  - Minimum values of domains satisfy constraints?
  - P_i = {3, 4, 5, 6}   as $3 + 3 + 3 + 3 = 12 \nleq 10$

- Domain restriction
  - Are the largest values consistent with the minimum ones?
  - P_i = {2, 3, 4, ~~5~~, ~~6~~} as $2 + 2 + 2 + (5\ or\ 6) = (11\ or\ 12) \nleq 10$

Range bounds

- Impractical to store large integer sets

- Ranges can be used [min, max] instead

- Bounds propagation can be used to restrict domains according to constraints

X domain [25, 100]	[75, 100]	$X+Y \geq 200$
Y domain [50, 125]	[100, 125]	

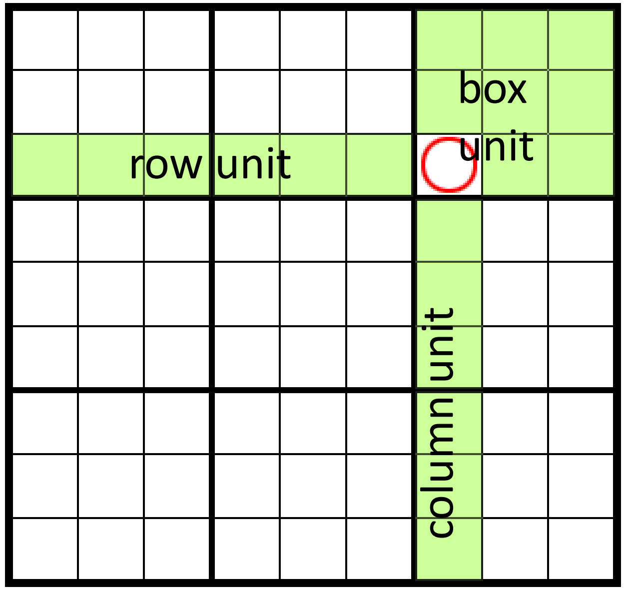
How did we get [75, 100]? $Y = 125 \rightarrow X \geq 75$

Maki Kaji 1951-2021 (photo: AP)

Sudoku

- Puzzle game played with digit symbols
- All-different constraints exist on *units*
- Some cells initially filled in

- Hard for humans, pretty simple for CSP solvers



Jef Vandenberghe Sodoku tutorial

Sudoku

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

Sudoku

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

Sample constraints

- Alldiff(A1,A2,A3,A4,A5,A6,A7,A8,A9)
- Alldiff(A1,B1,C1,D1,E1,F1,G1,H1,I1)
- Alldiff(A1,A2,A3,B1,B2,B3,C1,C2,C3)

These can be expanded to binary constraints, e.g. $A1 \neq A2$

Sudoku

AC-3 constraint propagation

- E6: d={1, 2, 3, 4, 5, 6, 7, 8, 9}
- Box constraints:
  d_1 = d - {1, 2, 7, 8} = {3, 4, 5, 6, 9}
- Column constraints:
  d_2 = d_1 – {2, 3, 5, 6, 8, 9} = {4}

Therefore E6=4

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

Sudoku

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7					4			8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

AC-3 constraint propagation

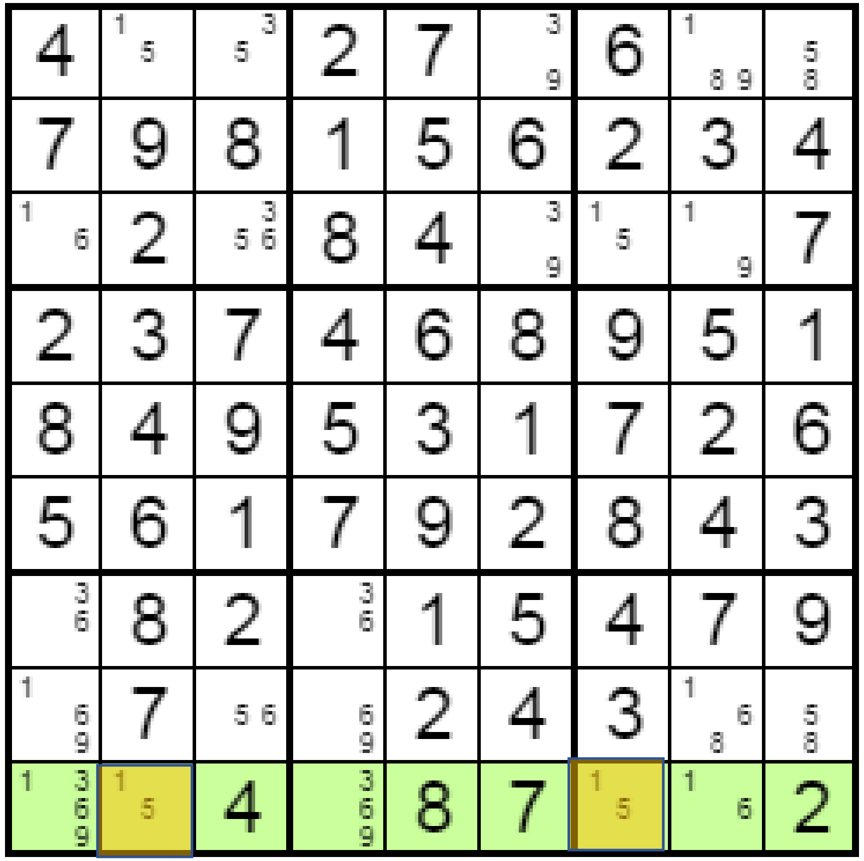
- I6: d={1, 2, 3, 4, 5, 6, 7, 8, 9}
- Column constraints:
  $d_1 = d - \{2, 3, 4, 5, 6, 8, 9\} = \{1, 7\}$
- Row constraints:
  $d_2 = d_1 - \{1, 3, 5\} = \{7\}$

Therefore I6=7

For this puzzle, continued application of AC-3 would solve the puzzle (not always true)

Naked sets

- Yellow squares form a *naked pair* {1, 5}
  - one must contain 1
  - other 5
- Can subtract 1 and 5 from domains of all other cells in row unit.
- These types of “tricks” are not limited to Sudoku puzzles.



Back to searching…

- Once all constraints have been propagated, search for a solution.
- Naïve search
  - Action picks a variable and a value. n variables domain size d → $n \cdot d$ possible search nodes
  - Search on next variable.
  - Backtrack when search fails.
- Problems with naïve search
  - n variables with domains of size d
  - nd choices for first variable, $(n-1)d$ for second….

$$nd \cdot (n-1)d \cdot \ldots \cdot 2d \cdot 1d = n!\, d^n$$

leaves but there are only d^n possible assignments!

Back to searching

- CSPs are commutative
- Order of variable selection does not affect correctness (may have other impacts)
- Modified search
  - Each level of search handles a specific variable.
  - Levels have d choices, leaving us with d^n leaves

Backtracking Search

```
def backtracking-search(CSP):
  return backtrack({}, CSP);  # call w/ no assignments

def backtrack(assignment, CSP):
  if all variables assigned, return assignment
  var = select-unassigned-variable(CSP, assignment)
  for each value in order-domain-values(var, assignment, csp):
    if value consistent with assignment:
      assignment.add({var = value})
      # propagate new constraints (will work without, but probably slowly)
      inferences = inference(CSP, var, assignment)
      if inferences ≠ failure:
        assignment.add(inferences)
        result = backtrack(assignment, CSP)
        if result ≠ failure, return result
    # either value inconsistent or further exploration failed
    # restore assignment to its state at top of loop and try next value
    assignment.remove({var = value}, inferences)
  # No value was consistent with the constraints
  return failure
```

Backtracking search

- Several strategies have been employed so far to make searches more efficient, e.g.
  - heuristics (best-first and A* search)
  - pruning (alpha-beta search)

- Can we come up with strategies to improve CSP search?

select-unassigned-variable

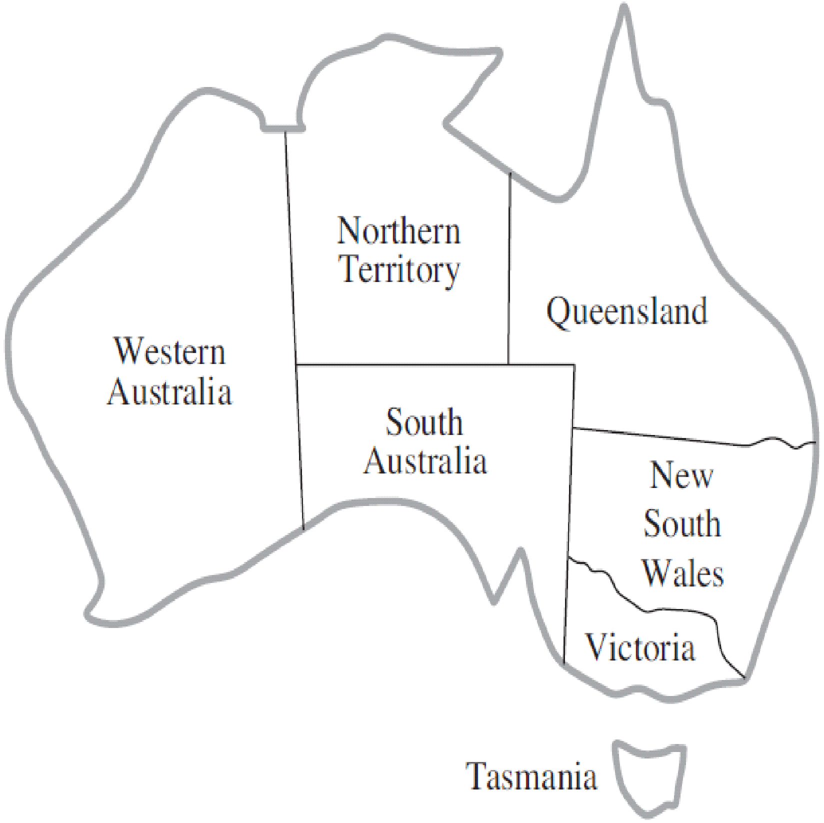
- Could try in order: $\{X_1, X_2, \ldots, X_n\}$
  Rarely efficient…

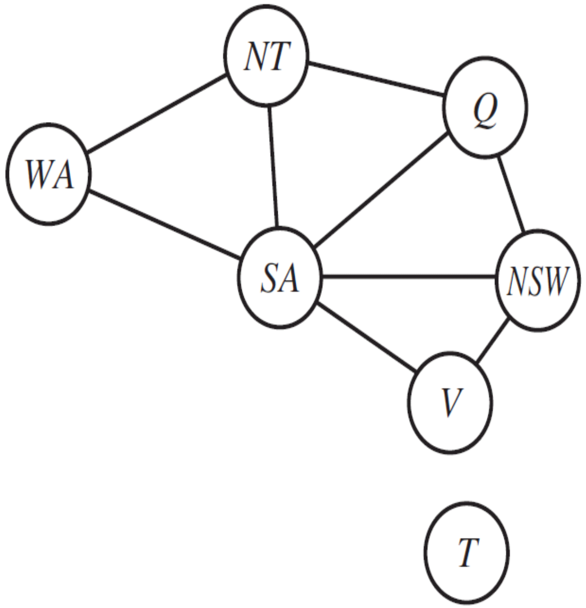
- Fail-first strategies
  - Minimum remaining value heuristic:
    Select the most constrained value; the one with the smallest domain.
    Rationale – probably the most likely variable to fail

  - Degree heuristic:
    Use the variable with the highest number of constraints on other unassigned variables.

select-unassigned-variable

- Minimum remaining value usually is a better performer than degree heuristic, but not always:





All variables have domains of size three at start, but degree of constraints differs.

order-domain-values

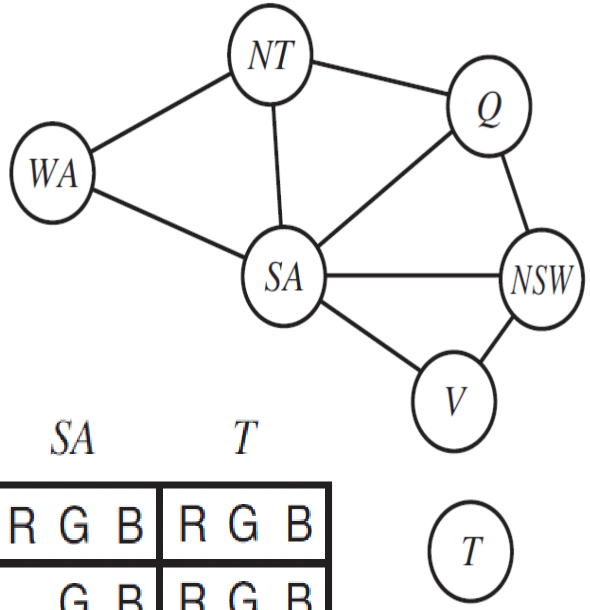
- The order of the values within a domain may or may not make a difference
- Order has no consequence
  - if goal is to produce all solutions or
  - if there are no solutions
- In other cases, we use a fail-last strategy
  - Pick the value that reduces neighbors' domains as little as possible.

Why fail-first for variable selection and fail-last for value selection?

inference in search

- forward-checking
  - Check arc consistency with neighboring variables.
  - We will see that maintaining arc-consistency (variant of AC3) is more powerful as it propagates all the way through the graph as opposed to forward checking that just looks at neighbors.

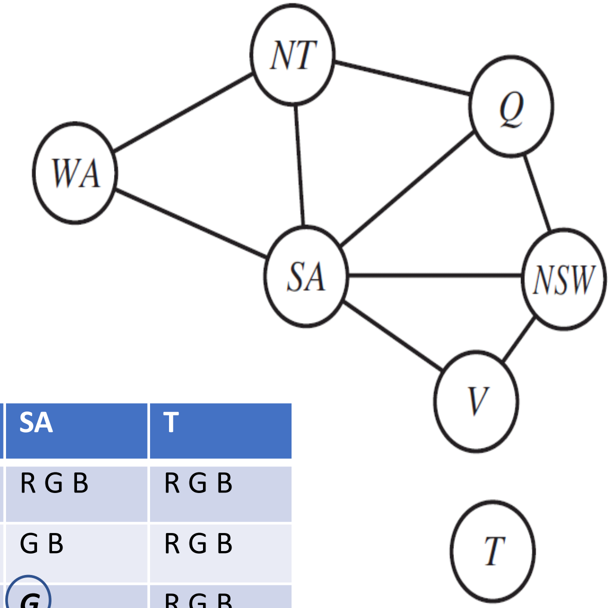
forward-checking example



	WA	NT	Q	NSW	V	SA	T
Initial domains	R G B	R G B	R G B	R G B	R G B	R G B	R G B
After *WA=red*	R	G B	R G B	R G B	R G B	G B	R G B
After *Q=green*	R	B	G	R B	R G B	B	R G B
After *V=blue*	R	B	G	R	B	~~X~~	R G B

Note: Variable selection is not by degree ordering or min. remaining value

forward-checking example



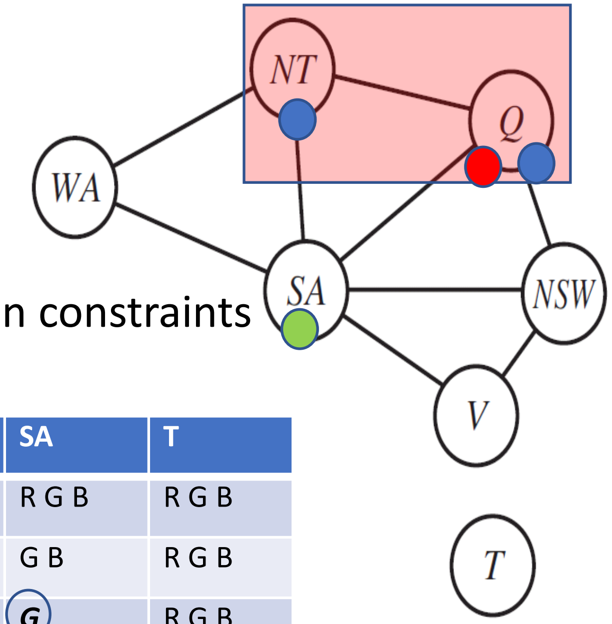
with minimum remaining value heuristic

	WA	NT	Q	NSW	V	SA	T
Initial domains	R G B	R G B	R G B	R G B	R G B	R G B	R G B
After WA=R	***R***	G B	R G B	R G B	R G B	G B	R G B
After SA=G	R	B	R B	R B	R B	***G***	R G B
After Q=R	R	B	***R***	B	R B	G	R G B
After V=R	R	B	R	B	***R***	G	R G B

forward-checking example

When we assigned SA=G, we restricted NT to B
However, Q was only restricted to R B

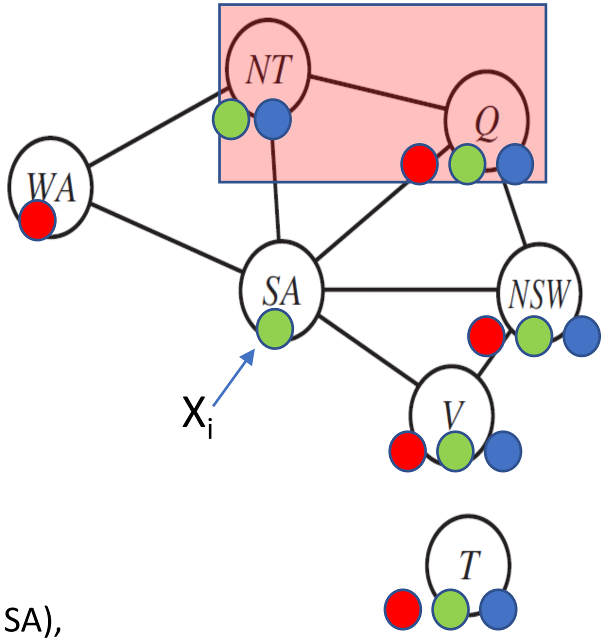
Forward checking does not check anything other than constraints with the neighbor being assigned.



	WA	NT	Q	NSW	V	SA	T
Initial domains	R G B	R G B	R G B	R G B	R G B	R G B	R G B
After WA=R	***R***	G B	R G B	R G B	R G B	G B	R G B
After SA=G	R	B	R B	R B	R B	***G***	R G B
After Q=R	R	B	***R***	B	R B	G	R G B
After V=R	R	B	R	B	***R***	G	R G B

Maintaining arc consistency (MAC)

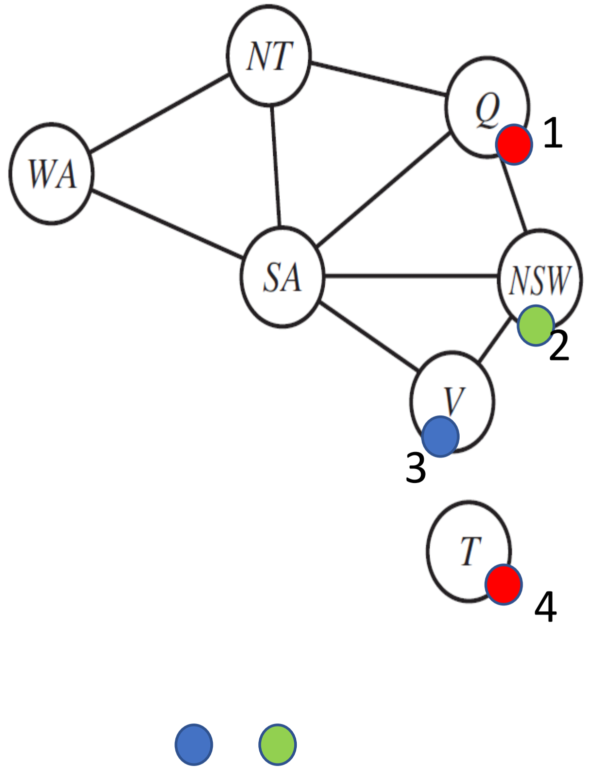
- Algorithm that propagates constraints beyond the node.
- AC3 algorithm with modified initial queue
  - typical AC3 – *all constraints*
  - MAC – constraints between selected variable X_i and its neighbors X_j
    $\{(X_j, X_i): X_j \text{ neighbor}(X_i)\}$



SA=green
queue: (NT, SA), (Q, SA),
(NSW, SA), (V, SA)
Process (NT, SA):
G removed from NT. Neighbors(NT)={WA,SA,Q}
enqueue (WA,NT) (Q,NT)
Process (Q, SA):
G removed from Q
enqueue (NT, Q), (NSW, Q)
and so on…

Intelligent backtracking

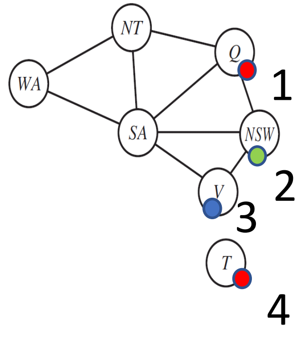
- Suppose variable ordering:
  Q, NSW, V, T, SA, WA, NT
- and assignments:
  {Q=red, NSW=green, V=blue, T=red}
- SA is problematic…
  - backtracking will try new values for Tasmania



- What if we could *backjump* to the variable that caused the problem?

Conflict-directed backjumping

- Maintain a *conflict set* for each variable X:
  A set of assignments that restricted values in X's domain.
- When a conflict occurs, we backtrack to the last conflict that was added.
- In the case of SA,
  - assignments to Q, NSW, and V restricted SA's domain
  - variable ordering: Q, NSW, V, T, SA, WA, NT
  - SA conflict set {Q=red,NSW=green,V=blue}
  - so we backjump to SA's last conflict V. with {Q=red,NSW=green}



Backjumping implementation

- On forward checks of X assigned to x,
  - when X deletes a value from Y's domain, add X=x to Y's conflict set
  - If Y is emptied, add Y's conflict set to X's and backjump to the last added conflict.
    Adding these lets us be smarter about where to backjump.

- Easy to implement, build conflict set during forward check.

Backjumping

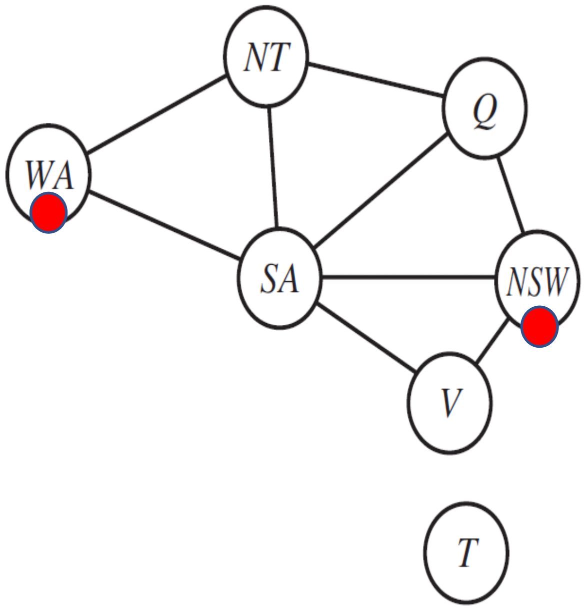
- What we prune in conflict-directed backjumping is redundant to what we'd prune from forward checking or MAC searches.
- Interesting, but better to just use forward checking/MAC…

- Still a good idea, what if we could extend it?

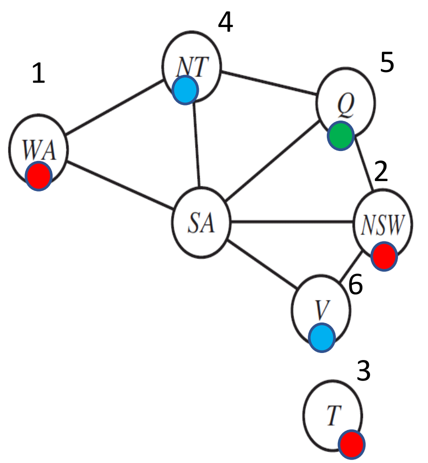
More sophisticated backjumps…

- Assignments to the right are inconsistent
  - Suppose we try and assign T, NT, Q, V, SA
  - SA, NT, Q have reduced domains {green, blue} and cannot be assigned
  - Backjumping fails when a domain is reduced to ∅ as SA, NT, and Q are consistent with WA, NSW.

- Can we determine that there is a *conflict set* {WA, SA, NT, Q} that are causing the issue?



Conflict-directed backjumps



- Variable order: WA, NSW, T, NT, Q, V, SA
- SA fails. conf(SA) = {WA=red, NT=blue, Q=green}
- Last variable in conf(SA) is Queensland
  - Absorb SA's conflict set into Q

$$conf(Q) = conf(Q) \cup conf(SA) - \{Q\}$$

  - conf(Q)
    = {NT=blue, NSW=red} U {WA=red, NT=blue,Q=green}-{Q=green}
    = {WA=red, NSW=red, NT=blue}

    Unable to assign a different color to Q, backjump
  - conf(NT) = conf(NT) U conf(Q)-{NT}

    = {WA=red} U {WA=red, NSW=red, NT=blue}

    = {WA=red, NSW=red}
    When we run out of colors for NT, we will backump to NSW

Note: conf(SA) would have had NSW=red if NSW was processed before WA

Constraint-learning and no-goods

- On the Australia CSP, we identified a minimal set of assignments that caused the problem.

- We call these assignment *no-goods.*

- We can avoid running into this problem again by adding a new constraint (or checking a no-good cache).

Local Search CSPs

- Alternative to what we have seen so far
- Assign everything at once
- Search changes one variable at a time
  - Which variable?

Min-Conflicts Local Search

```
def minconflicts(csp, maxsteps):
  current = assign all variables
  for i = 1 to maxsteps:
    if solution(current), return current
    var = select conflicted variable at random from current
    val = find value that minimizes the number of conflicts
    update current such that var=val
  return failure
```

Min-Conflicts local search

- Pretty effective for many problems, e.g.
  million queens problem can be solved in about 50 steps
- This is essentially a greedy search, consequently:
  - local extrema
  - can plateau
  - many techniques discussed for hill climbing can be applied (e.g. simulated annealing, plateau search)